Efficiently Embedding QUBO Problems on Adiabatic Quantum Computers

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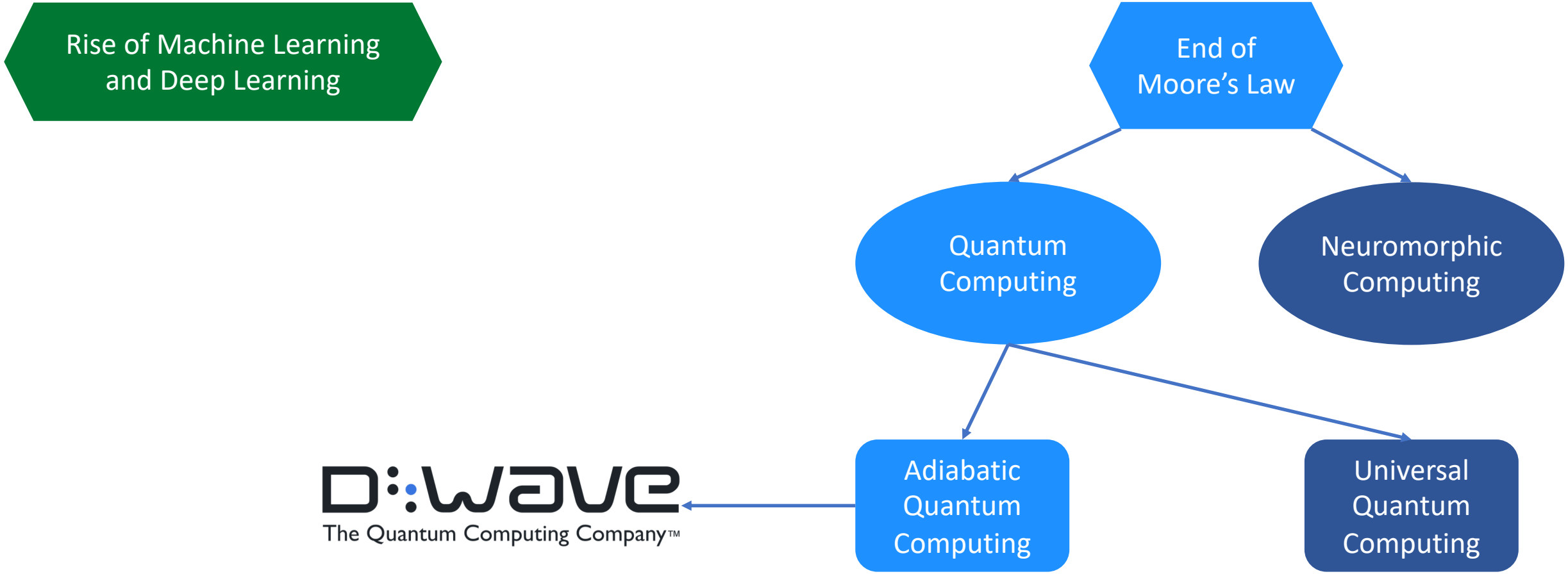
Quantum Computing with D-Wave



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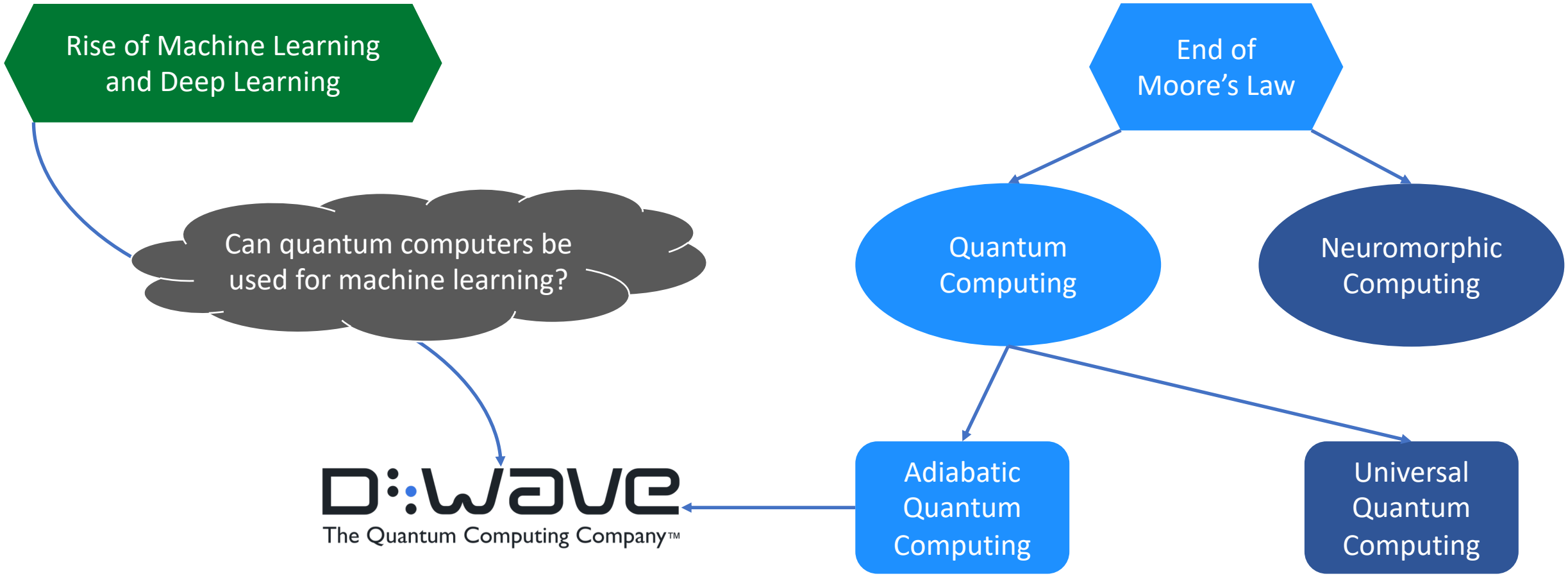

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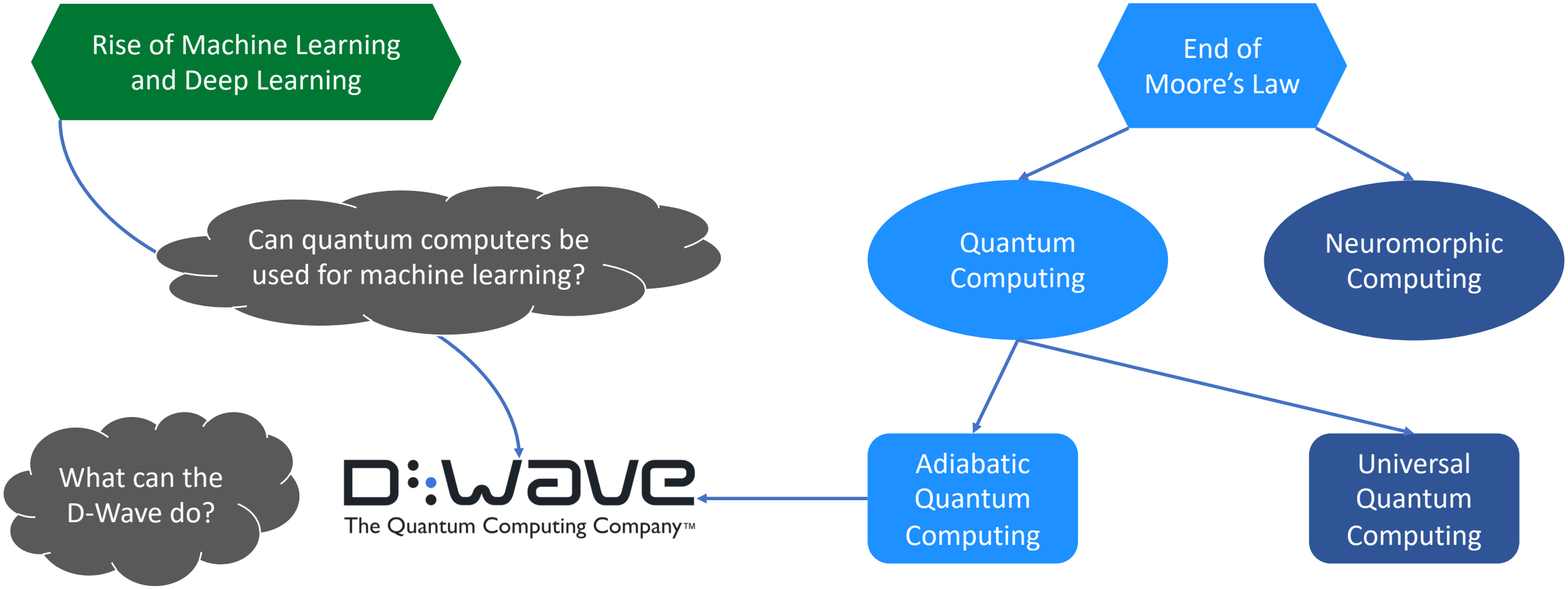
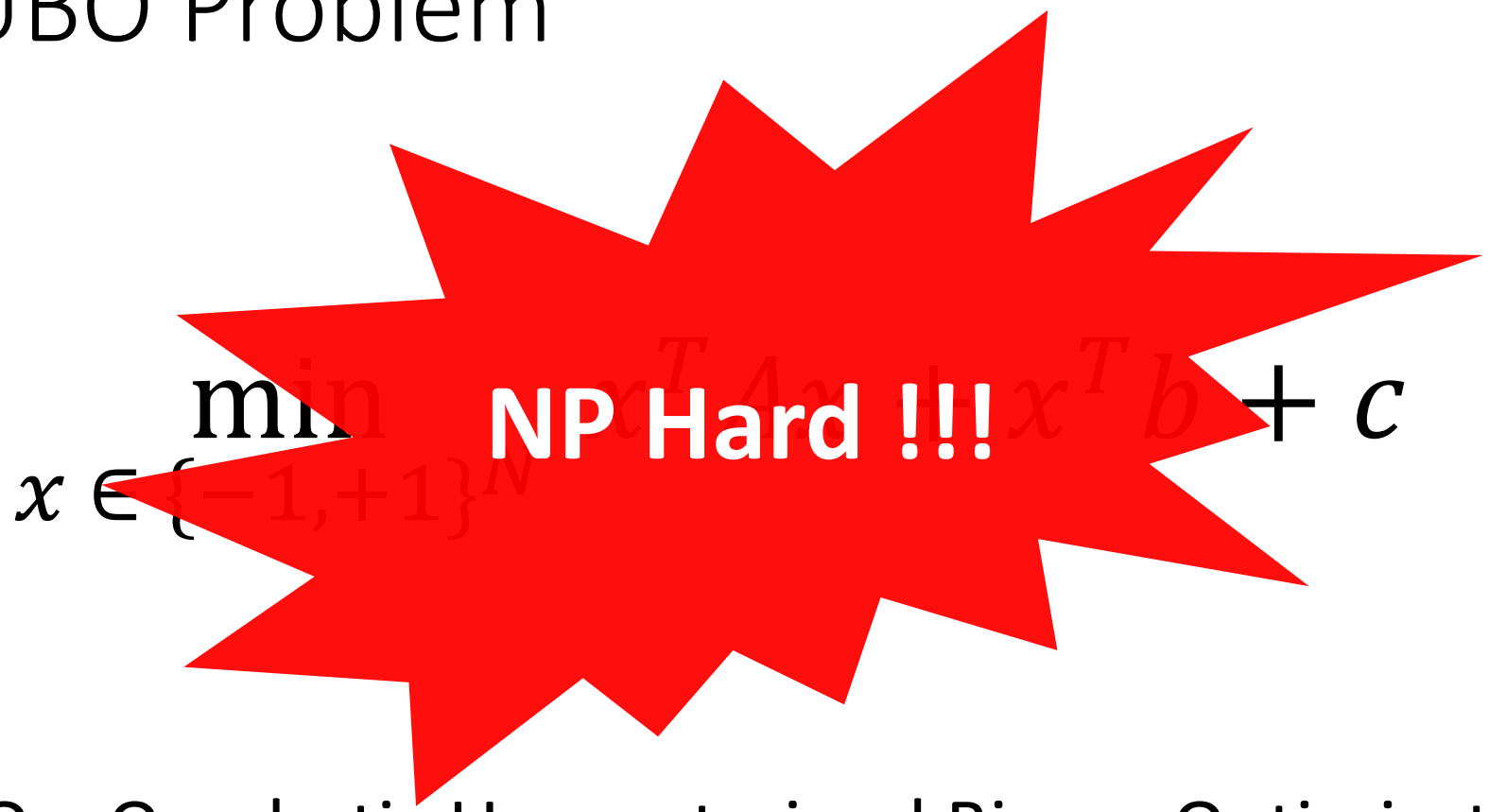


The QUBO Problem

$$\min_{x \in \{-1,+1\}^N} x^T A x + x^T b + c$$

QUBO = Quadratic Unconstrained Binary Optimization

The QUBO Problem



$$\min_{x \in \{-1,+1\}^N} x^T A x + x^T b + c$$

QUBO = Quadratic Unconstrained Binary Optimization

Embedding

What is Embedding?

Traveling Salesman Problem (TSP)

Airline Scheduling Problem

Protein Folding Problem

Node Packing Problem

⋮

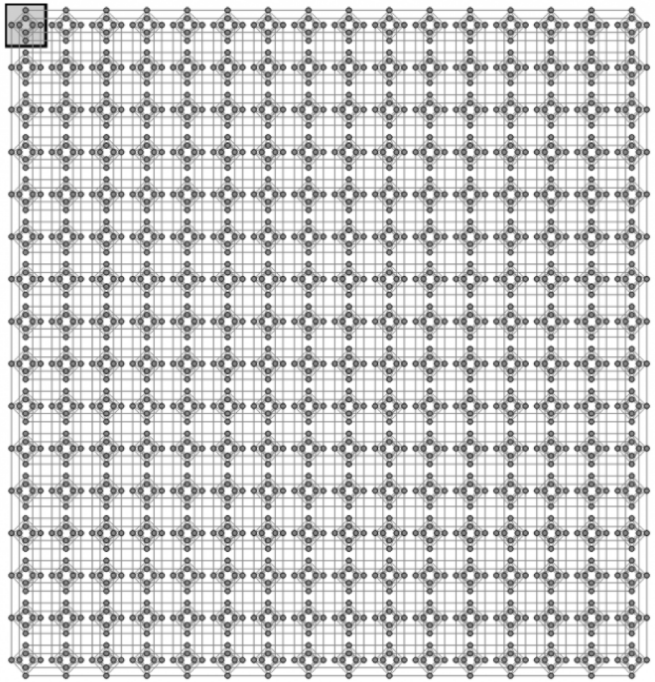
Any NP Hard Problem

QUBO Problem

EMBEDDING

D-Wave Hardware

$$\min_{x \in \{-1,+1\}^N} x^T A x + x^T b + c$$



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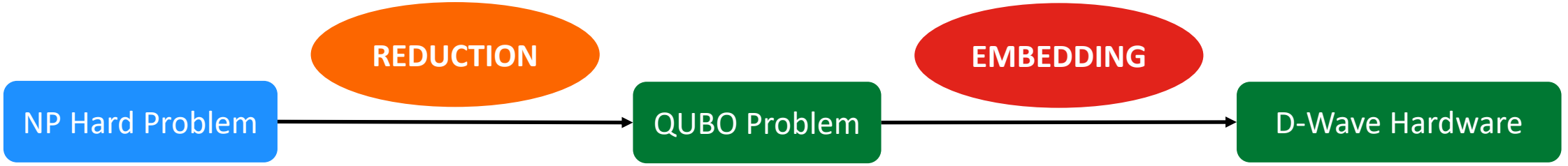

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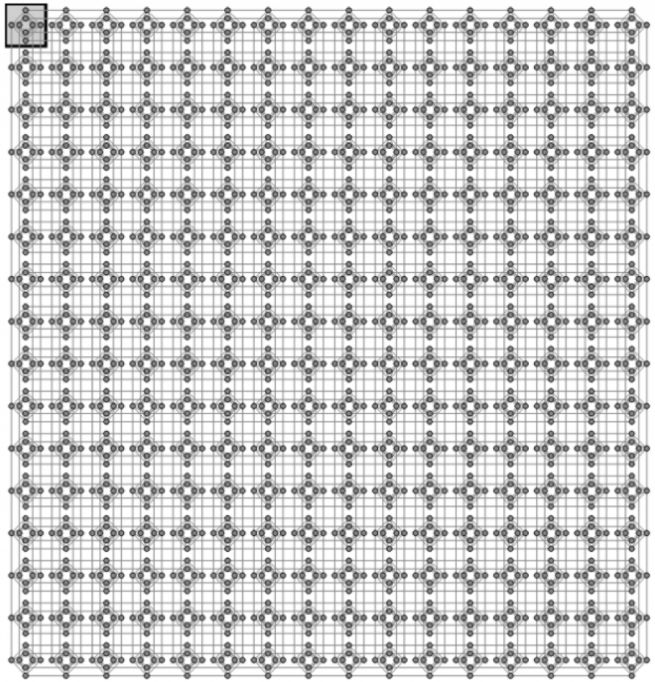
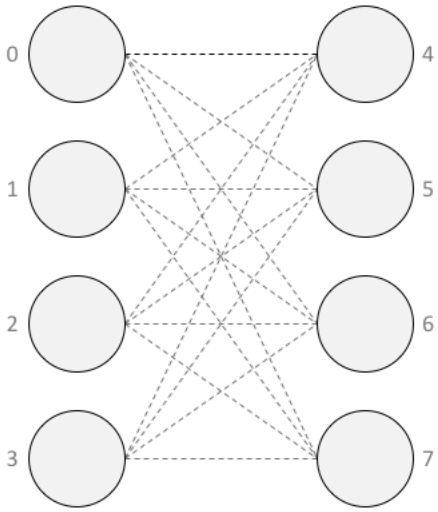


Why is Embedding hard?



$$\begin{bmatrix} 0 & 12 \\ 0 & 0 \end{bmatrix}$$

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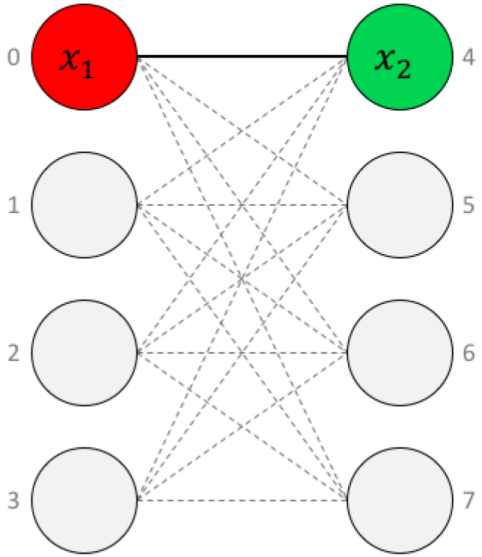

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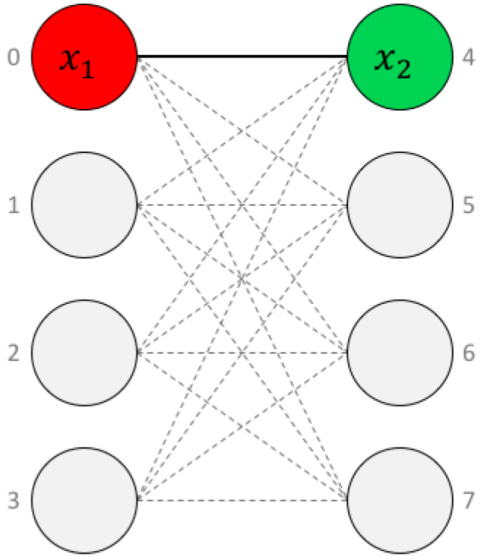

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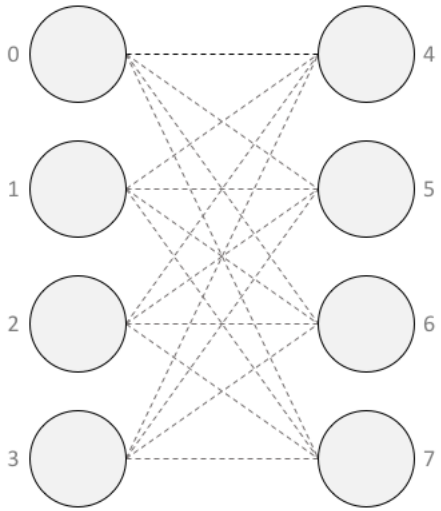

$$\begin{bmatrix} 0 & 12 & 13 \\ 0 & 0 & 21 \\ 0 & 0 & 0 \end{bmatrix}$$

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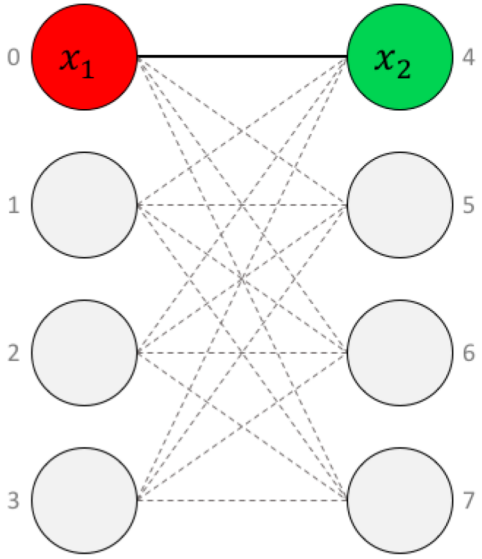

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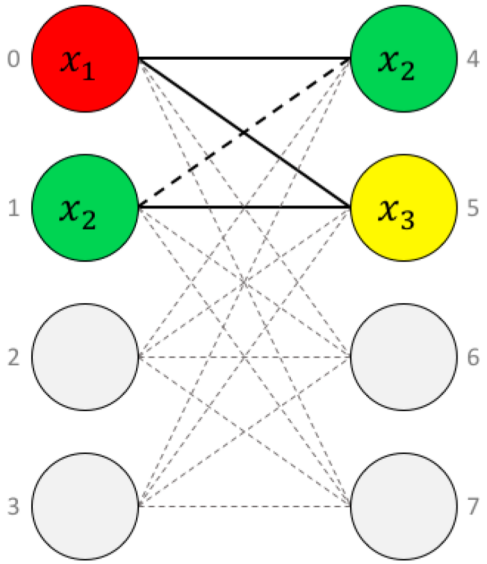

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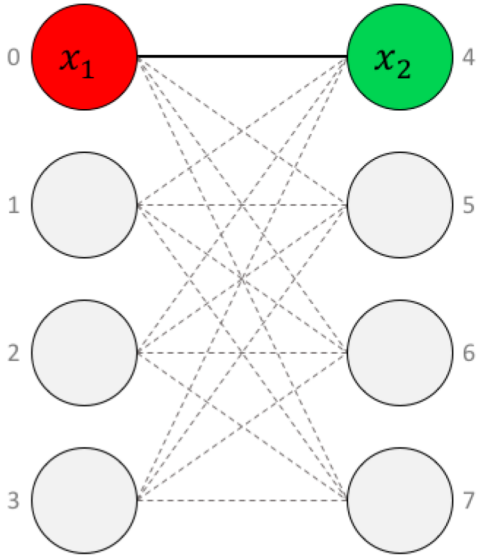

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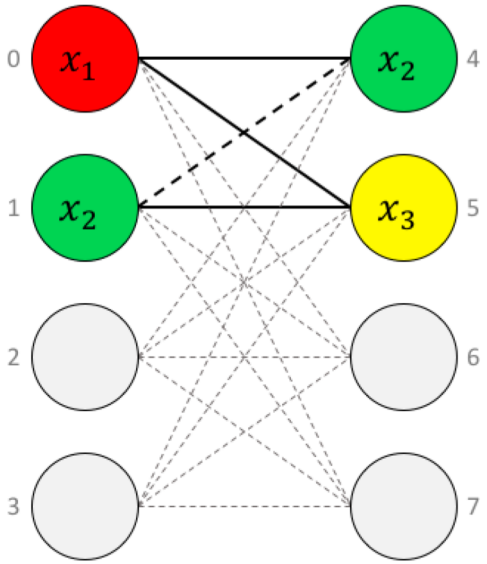

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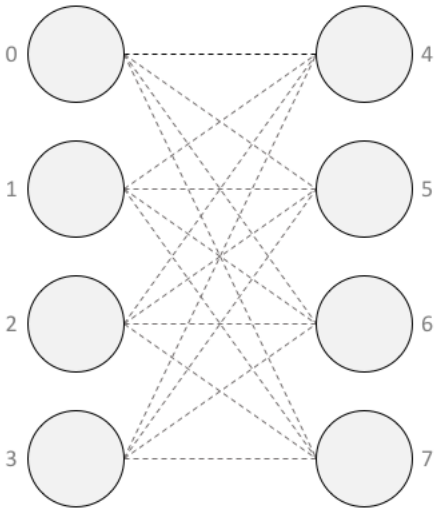

$$\begin{bmatrix} 0 & 12 & 13 & 14 \\ 0 & 0 & 23 & 24 \\ 0 & 0 & 0 & 34 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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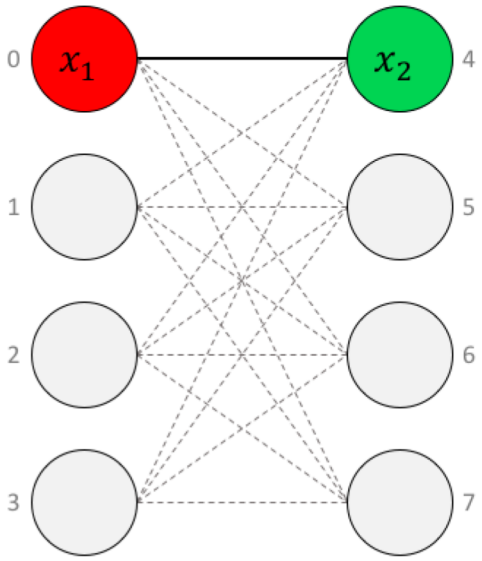

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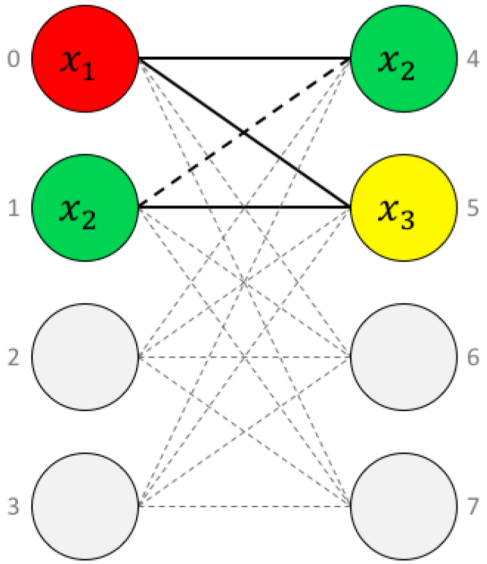

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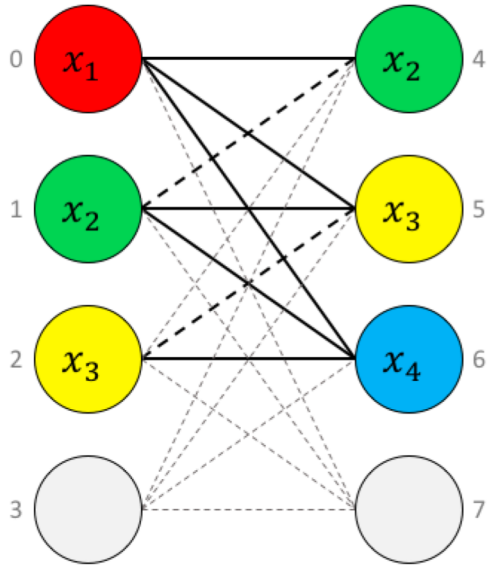

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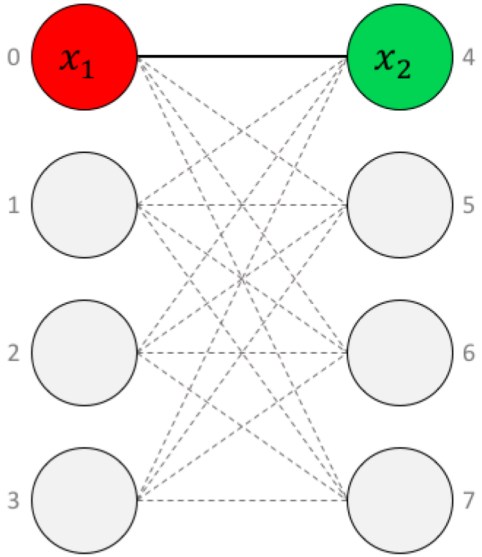

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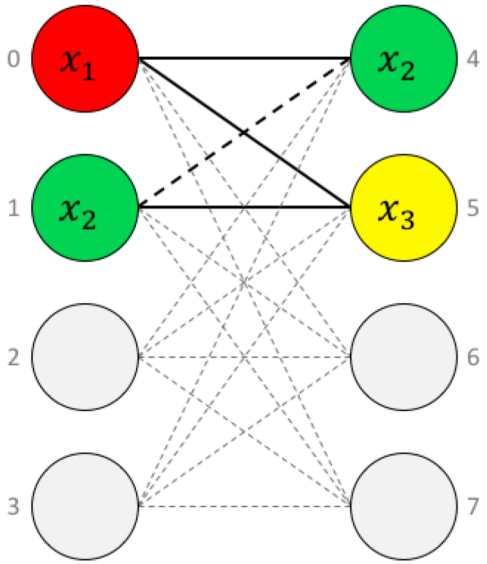

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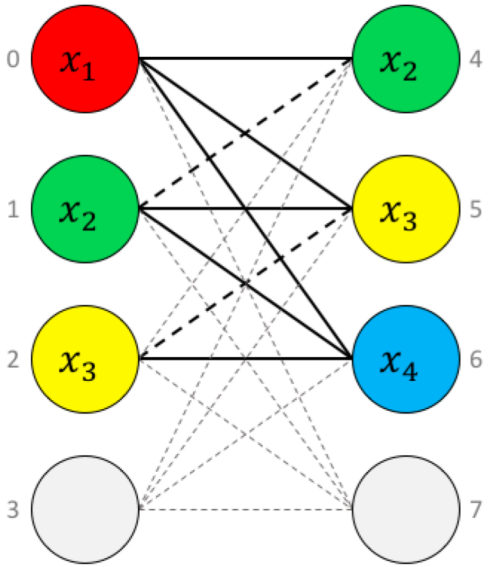

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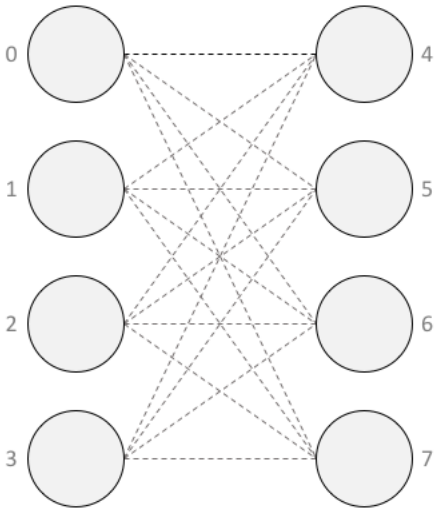

$$\begin{bmatrix} 0 & 12 & 13 & 14 & 15 \\ 0 & 0 & 23 & 24 & 25 \\ 0 & 0 & 0 & 34 & 35 \\ 0 & 0 & 0 & 0 & 45 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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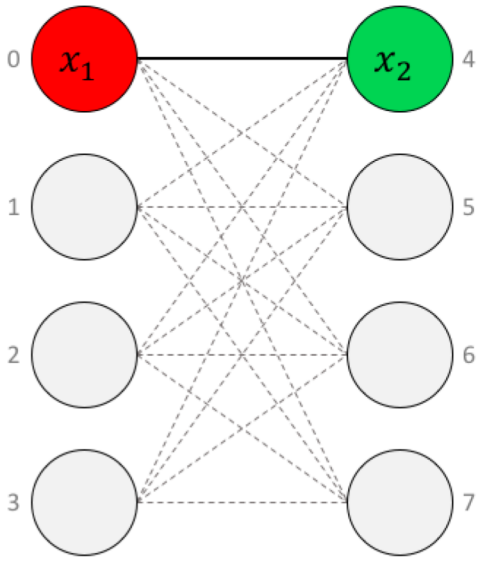

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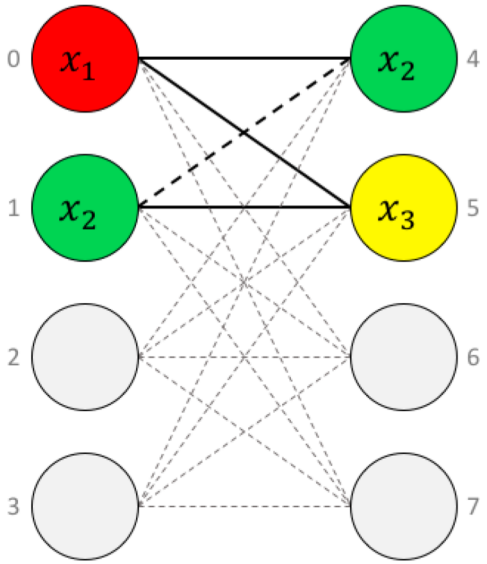

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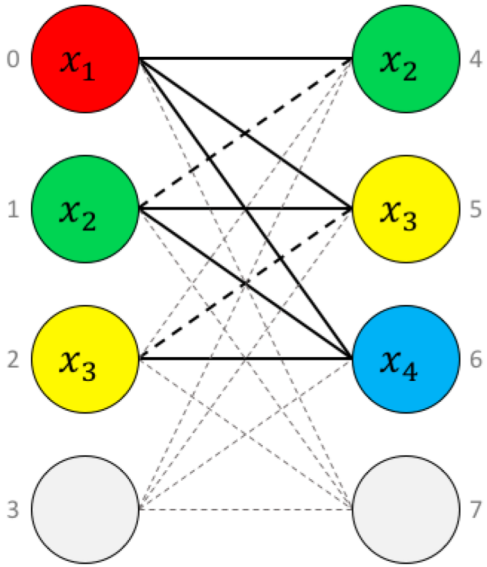

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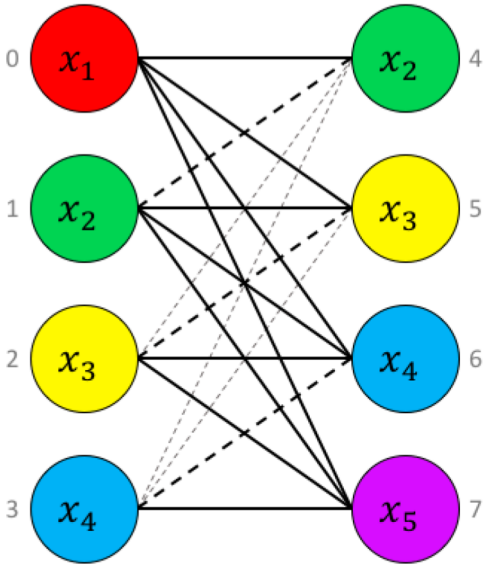

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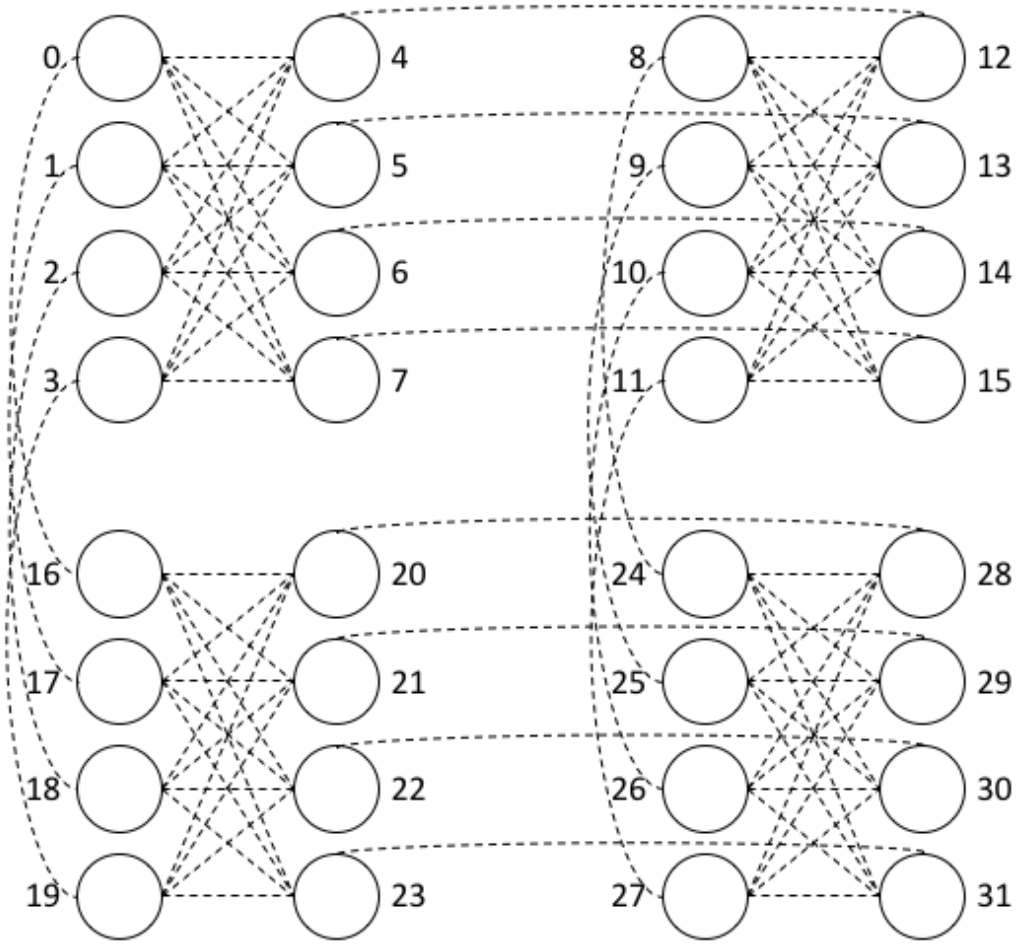
Why is Embedding hard?

$$A = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{matrix} \\ \begin{bmatrix} 0 & 12 & 13 & 14 & 15 & 16 \\ 0 & 0 & 23 & 24 & 25 & 26 \\ 0 & 0 & 0 & 34 & 35 & 36 \\ 0 & 0 & 0 & 0 & 45 & 46 \\ 0 & 0 & 0 & 0 & 0 & 56 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{matrix}$$



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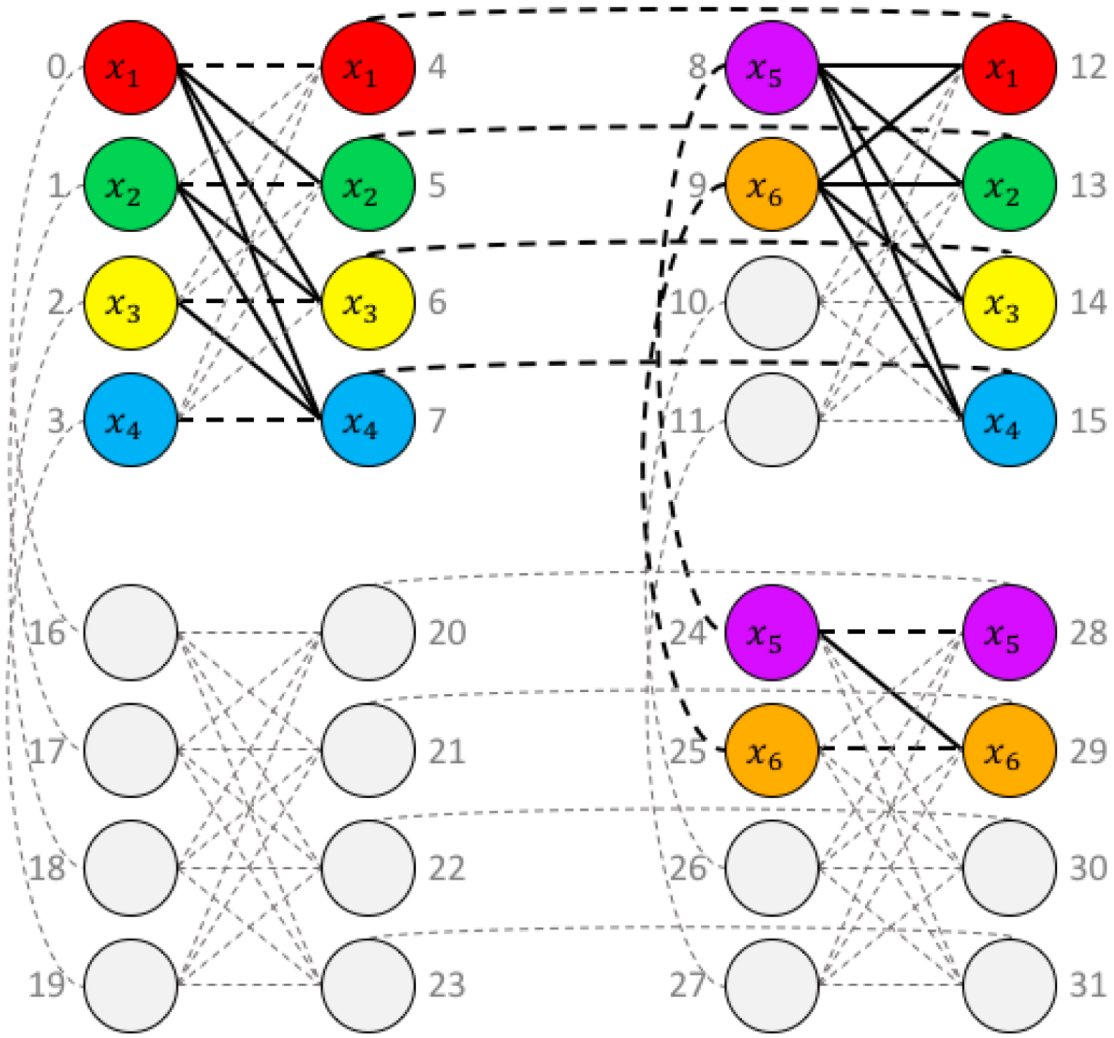


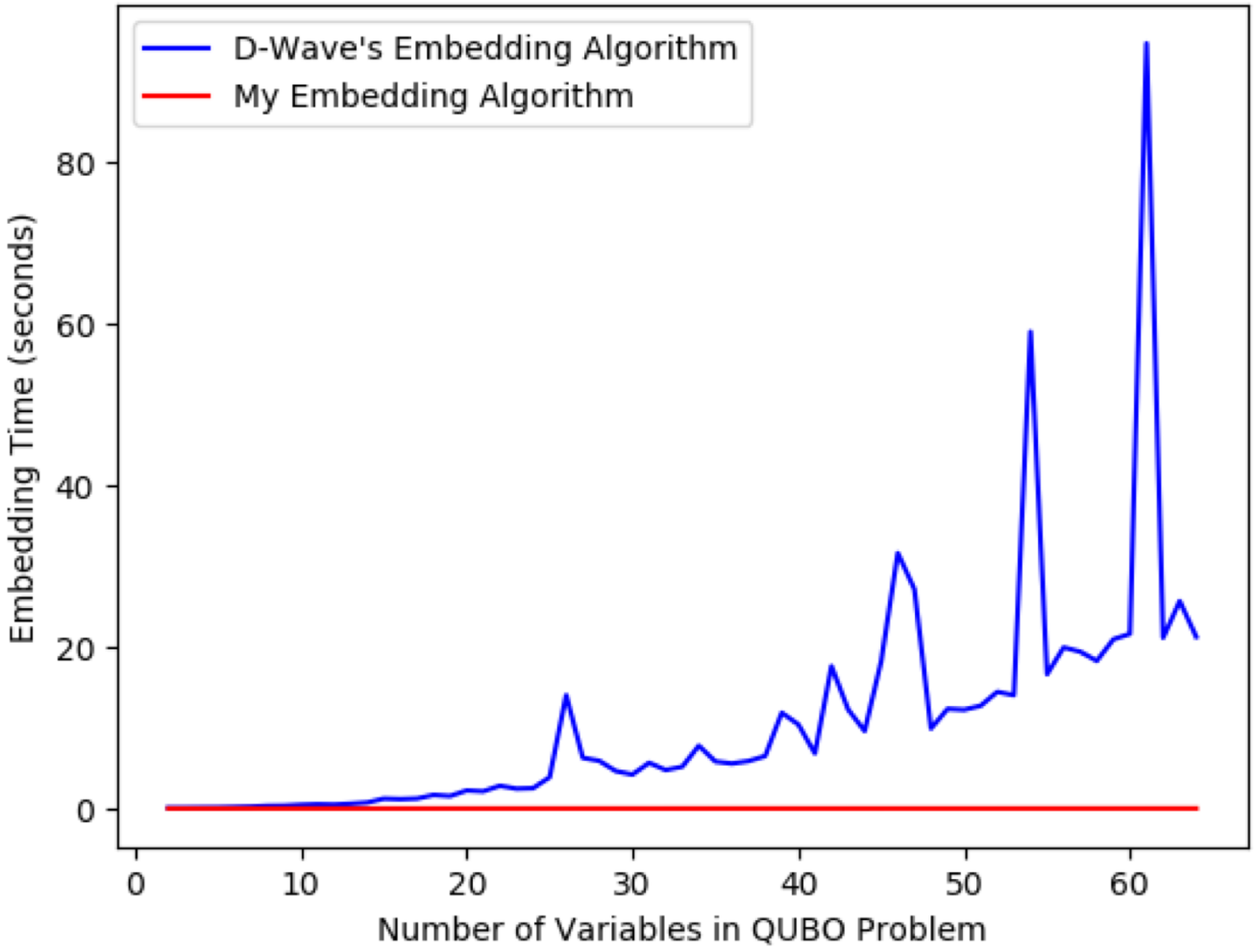
What makes a GOOD Embedding Algorithm?

- **Time**: Should run as fast as possible
- **Qubit Footprint**: Should as few qubits as possible
- **Accuracy**: Should get the objective function value as close to the global minima as possible

Embedding Algorithm Comparison

- Compared my algorithm to D-Wave's algorithm
- Criteria for comparison:
  - Embedding Time
  - Qubit Footprint
  - Accuracy
- Generated problems synthetically, while ensuring at least one global minima

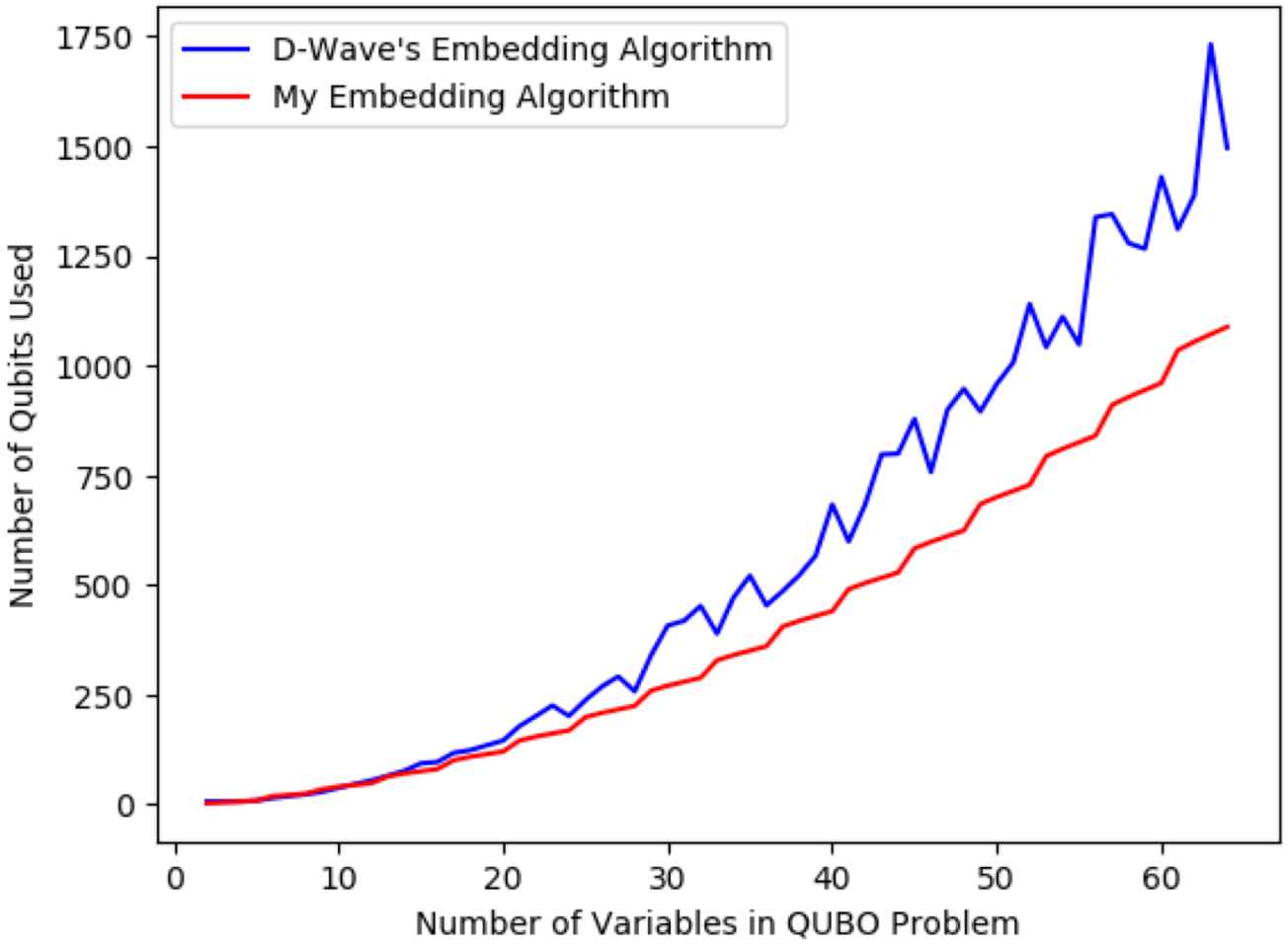
Embedding Algorithm Comparison: Time



QUBO Problem:

$$\min_{x \in \{-1,+1\}^N} x^T A x + x^T b + c$$

Embedding Algorithm Comparison: Qubit Footprint

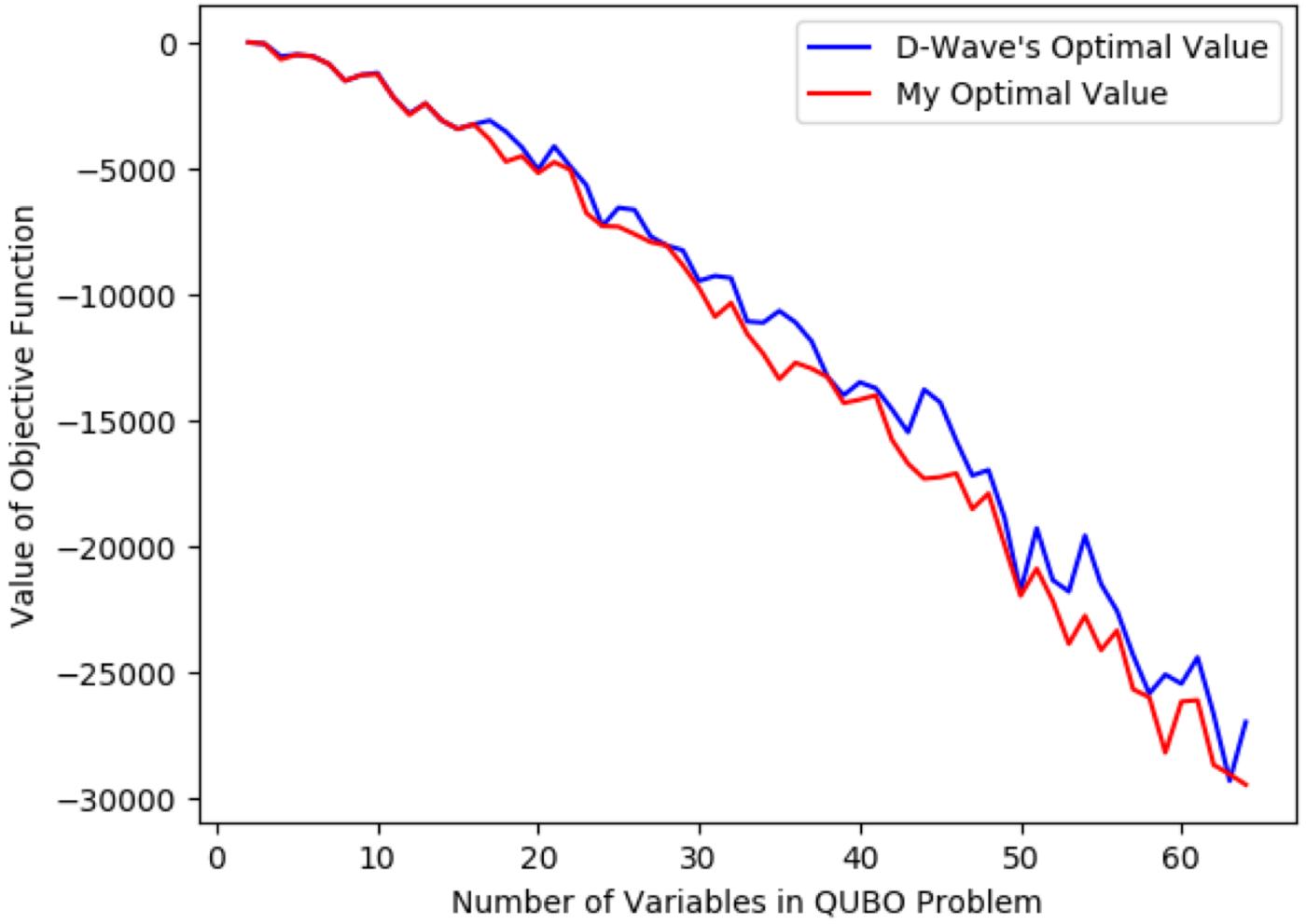


30% Less Qubits!!!

QUBO Problem:

$$\min_{x \in \{-1,+1\}^N} x^T A x + x^T b + c$$

Embedding Algorithm Comparison: Accuracy



QUBO Problem:

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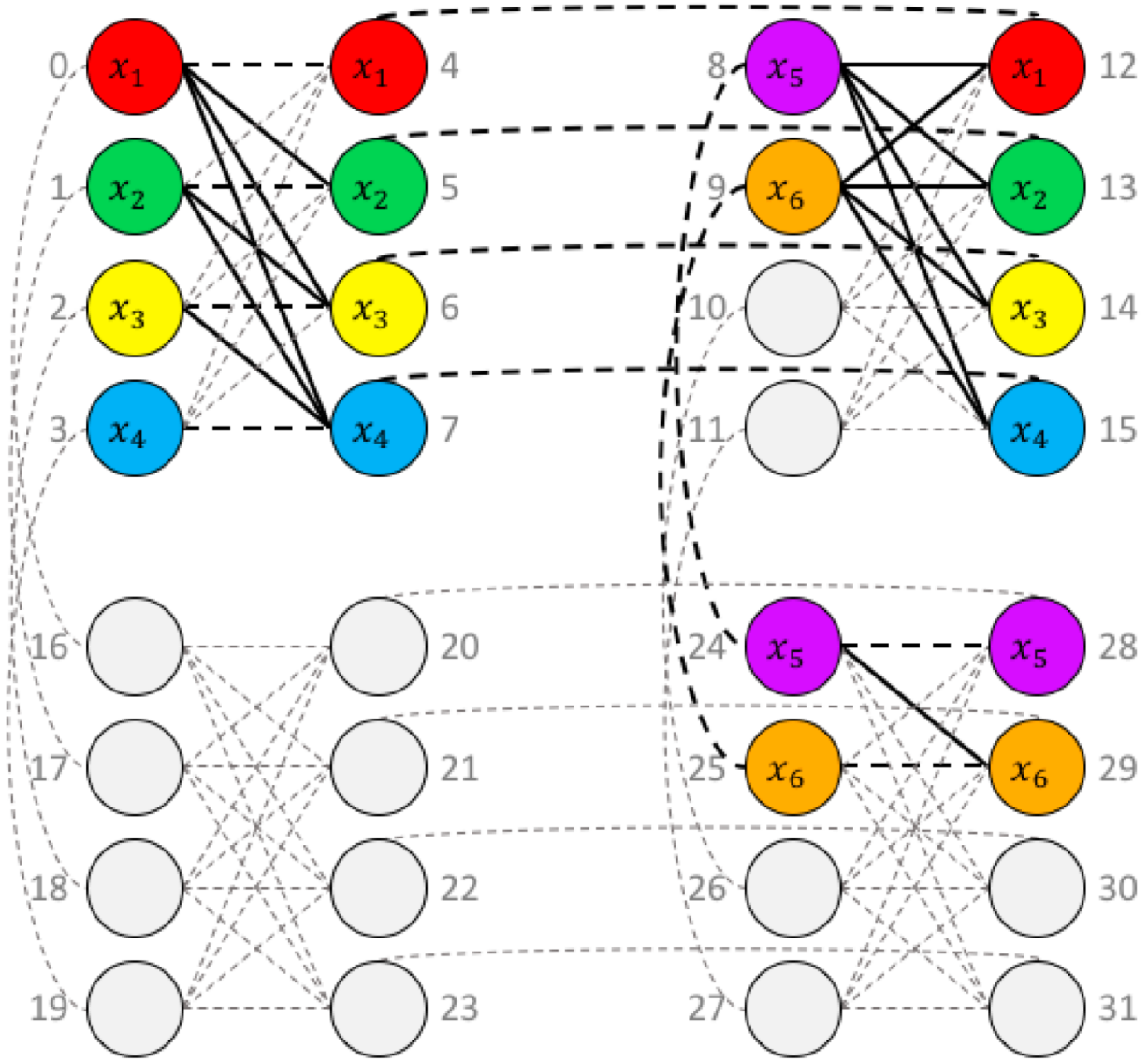
Other Metrics

- Longest qubit chain length
- Number of bit flips from globally optimal solution



Conclusion

- Quantum Computing with D-Wave
- Efficient Embedding Algorithm for D-Wave
- Other Work: Quantum Machine Learning

Thank You!

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